Summer Packet for Students Entering 12th Grade Applications and Interpretations SL

Directions:

1. Please complete all questions in the work space provided

2. You must show all of your work for each question. If you are stuck, please use the resource links provided in the section.
3. Your teacher will check this assignment on the FIRST day of school at the beginning of class. This will be your first completion grade so make sure you have attempted each problem.
4. If there are questions that you had a difficult time with, please list them in the box below (or highlight them on a separate sheet of paper). We expect you to use the resources provided if you are stuck, but understand there may be additional support needed for some questions.

Assessment:

1. On the second day of math, you will have a summative quiz based on the skills on this summer packet.
2. You will get 5 points for bringing your TI-84 CE Plus to class with you on the day of the test. This is a required tool that you will use throughout high school.

Extra Support:

1. The math department will have extra help days for the summer packet close to the return of school. Please check the school's website during the summer for the dates.
2. On the first day back to school, we will dedicate time in class to go over the answers and for you to ask your teacher questions.

12th Summer Review Math Applications and Interpretations

Please show all required work of each of the following questions.

A formula book and calculator are permitted on this assignment

Completed problems are due on the first day of class. Please attend scheduled help session if needed.

On paper 1 problems (# 1- 15) show work in provided space and answers in answer blank

Please work problems # 16- 20 on provided paper 2.

Each problem should begin a new page. Extra paper 2 is included. Leave a blank line between each section.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Answers must be written within the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. A group of 20 students travelled to a gymnastics tournament together. Their ages, in years, are given in the following table.

Age (years)	14	15	16	17	18	19	20	22
Frequency	1	2	7	1	4	1	1	3

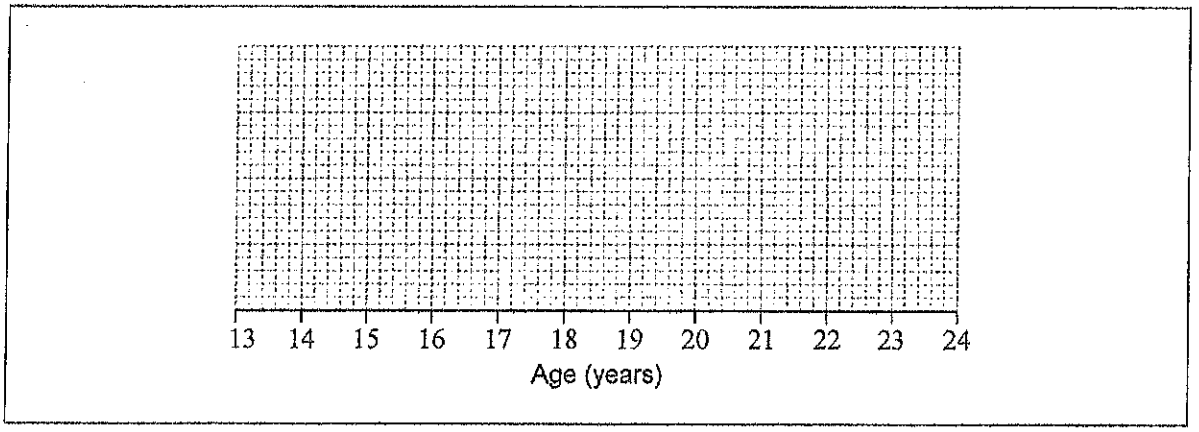
(a) For the students in this group

(i) find the mean age;

(ii) write down the median age. [3]

The lower quartile of the ages is 16 and the upper quartile is 18.5.

(b) Draw a box-and-whisker diagram, for these students' ages, on the following grid. [3]



Working:

Answers:

(a) (i) .....

(ii) .....

2. A class of 13 Mathematics students received the following grades in their final IB examination.

3 5 3 4 7 3 2 7 5 6 5 3 4

For these grades, find

(a) the mode; *[1]*

(b) the median; *[2]*

(c) the upper quartile; *[1]*

(d) the interquartile range. *[2]*

Working:

Answers:

(a) .....

(b) .....

(c) .....

(d) .....

3. In a particular week, the number of eggs laid by each hen on a farm was counted. The results are summarized in the following table.

Number of eggs	1	2	3	4	5	6
Frequency	4	7	12	10	14	13

(a) State whether these data are discrete or continuous. *[1]*

(b) Write down

(i) the number of hens on the farm;

(ii) the modal number of eggs laid. *[2]*

(c) Calculate

(i) the mean number of eggs laid;

(ii) the standard deviation. *[3]*

Working:

Answers:

(a) .....

(b) (i) .....

(ii) .....

(c) (i) .....

(ii) .....

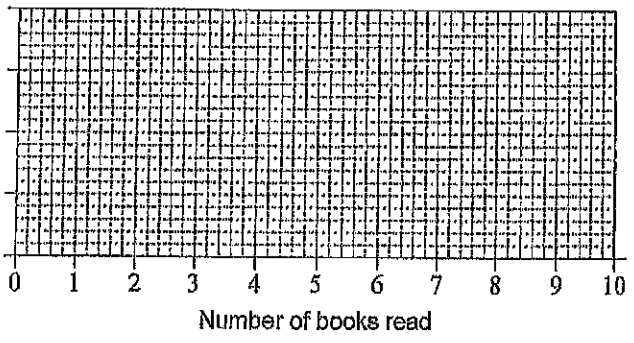
4. Two groups of 40 students were asked how many books they have read in the last two months. The results for the first group are shown in the following table.

Number of books read	Frequency
2	5
3	8
4	13
5	7
6	4
7	2
8	1

The quartiles for these results are 3 and 5.

(a) Write down the value of the median for these results. [1]

(b) Draw a box-and-whisker diagram for these results on the following grid. [3]



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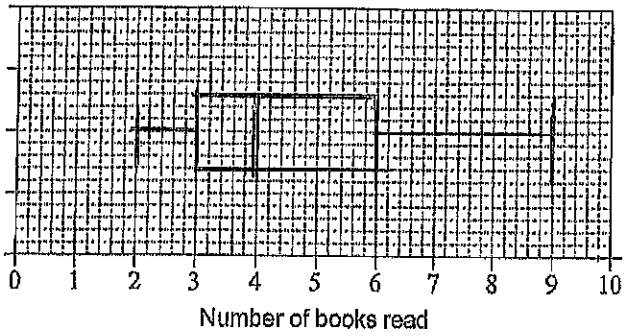
(Question 4 continued)

The results for **the second group** of 40 students are shown in the following box-and-whisker diagram.



(c) Estimate the number of students **in the second group** who have read at least 6 books. [2]

Working:

Answers:

(a) .....

(c) .....

5. Mr Burke teaches a mathematics class with 15 students. In this class there are 6 female students and 9 male students.

Each day Mr Burke randomly chooses one student to answer a homework question.

(a) Find the probability that on any given day Mr Burke chooses a female student to answer a question. [1]

In the first month, Mr Burke will teach his class 20 times.

(b) Find the probability he will choose a female student 8 times. [2]

(c) Find the probability he will choose a male student at most 9 times. [3]

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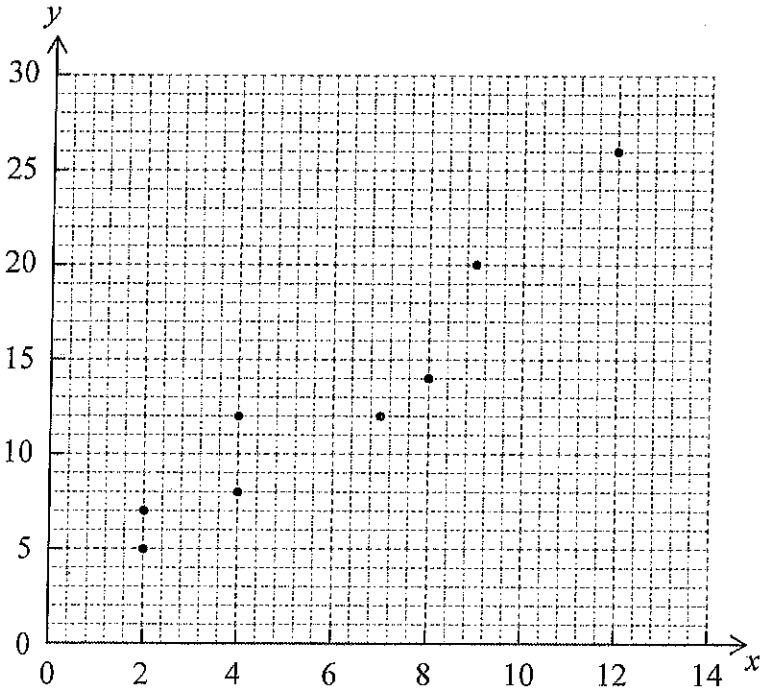
6. Consider the following set of data which is plotted on the scatter diagram below.

x	2	4	7	12	4	8	9	2
y	5	8	12	26	12	14	20	7



(a) Write down the coordinates of the mean point $(\bar{x}, \bar{y})$. *[2 marks]*

(b) Write down the value of r, the Pearson's product-moment correlation coefficient for this set of data. *[2 marks]*

(c) Draw the regression line for y on x on the set of axes above. *[2 marks]*

Working:

Answers:

(a) .....

(b) .....

7, Minta deposits 1000 euros in a bank account. The bank pays a nominal annual interest rate of 5 %, **compounded quarterly**.

(a) Find the amount of money that Minta will have in the bank after 3 years. Give your answer correct to two decimal places. [3]

Minta will withdraw the money from her bank account when the interest earned is 300 euros.

(b) Find the time, in years, until Minta withdraws the money from her bank account. [3]

Working:

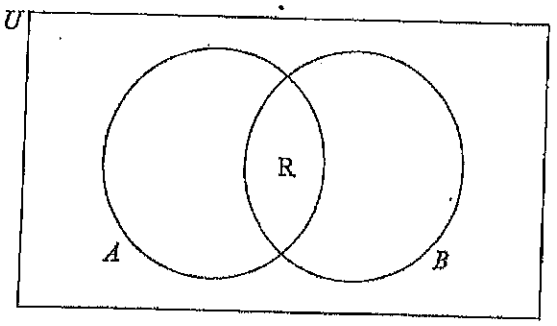
Answers:

(a) .....

(b) .....

8. Tuti has the following polygons to classify: rectangle (R), rhombus (H), isosceles triangle (I), regular pentagon (P), and scalene triangle (T).

In the Venn diagram below, set A consists of the polygons that have at least one pair of parallel sides, and set B consists of the polygons that have at least one pair of equal sides.



(a) Complete the Venn diagram by placing the letter corresponding to each polygon in the appropriate region. For example, R has already been placed, and represents the rectangle. *[3]*

(b) State which polygons from Tuti's list are elements of

(i) $A \cap B$;

(ii) $(A \cup B)'$.

[3]

Working:

Answers:

(b) (i) .....

.....

.....

(ii) .....

.....

.....

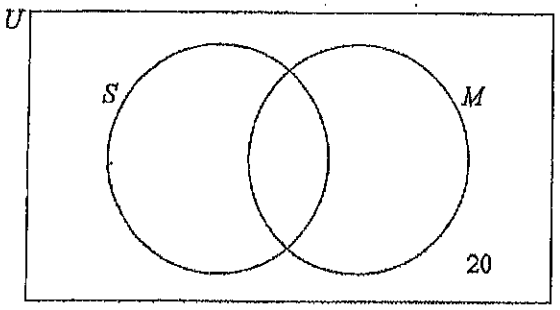
9,

Rosewood College has 120 students. The students can join the sports club (S) and the music club (M).

For a student chosen at random from these 120, the probability that they joined both clubs is $\frac{1}{4}$ and the probability that they joined the music club is $\frac{1}{3}$.

There are 20 students that did not join either club.

(a) Complete the Venn diagram for these students. [2]



(b) One of the students who joined the sports club is chosen at random.
Find the probability that this student joined both clubs. [2]

(c) Determine whether the events S and M are independent. [2]

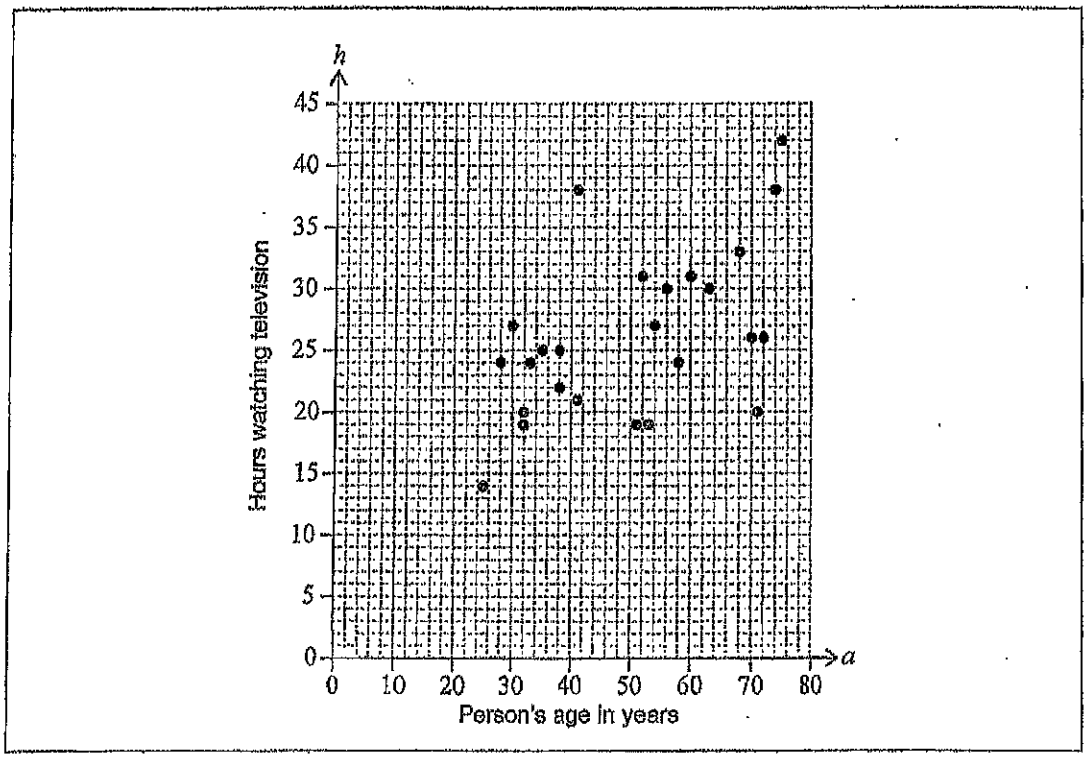
Working:

Answers:

(b) .....

(c) .....

10. A survey was carried out to investigate the relationship between a person's age in years (a) and the number of hours they watch television per week (h). The scatter diagram represents the results of the survey.



The mean age of the people surveyed was 50.

For these results, the equation of the regression line h on a is $h = 0.22a + 15$.

(a) Find the mean number of hours that the people surveyed watch television per week. [2]

(b) Draw the regression line on the scatter diagram. [2]

(c) By placing a tick (✓) in the correct box, determine which of the following statements is true:

The correlation between h and a is positive.	
The correlation between h and a is negative.	
There is no correlation between h and a.	

[1]

(d) Diogo is 18 years old. Give a reason why the regression line should not be used to estimate the number of hours Diogo watches television per week. [1]

(This question continues on the following page)

(Question . continued)

Working:

Answers:

(a) .....

(d) .....

.....

11. Yun Bin invests 5000 euros in an account which pays a nominal annual interest rate of 6.25 %, **compounded monthly.**
Give all answers correct to two decimal places.

Find

(a) the value of the investment after 3 years; *[3 marks]*

(b) the difference in the final value of the investment if the interest was compounded quarterly at the same nominal rate. *[3 marks]*

Working:

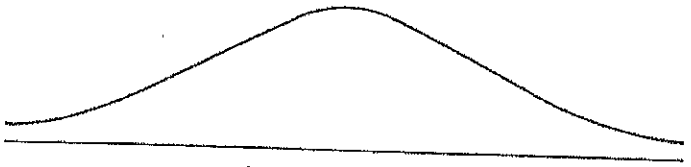
Answers:

(a) .....

(b) .....

12. A factory makes metal bars. Their lengths are assumed to be normally distributed with a mean of 180 cm and a standard deviation of 5 cm.

(a) On the following diagram, shade the region representing the probability that a metal bar, chosen at random, will have a length less than 175 cm. [2]



Length of metal bar

A metal bar is chosen at random.

(b) (i) The probability that the length of the metal bar is less than 175 cm is equal to the probability that the length is greater than h cm. Write down the value of h.

(ii) Find the probability that the length of the metal bar is greater than one standard deviation above the mean. [4]

Working:

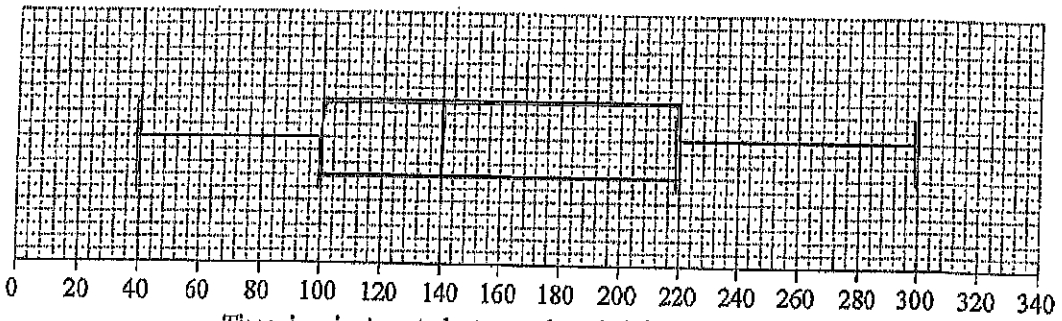
Answers:

(b) (i) .....

(ii) .....

13. *Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.*

The time, in minutes, that students in a school spend on their homework per day is presented in the following box-and-whisker diagram.



Time, in minutes, students spend on their homework per day

(a) Find

(i) the longest amount of time spent on homework per day;

(ii) the interquartile range. *[3]*

(b) State the statistical term corresponding to the value of 140 minutes. *[1]*

(c) Find the percentage of students who spend

(i) between 100 and 140 minutes per day on their homework;

(ii) more than 100 minutes per day on their homework. *[2]*

(This question continues on the following page)

(Question : continued)

Working:

Answers:

(a) (i) .....

(ii) .....

(b) .....

(c) (i) .....

(ii) .....

14. The number of apartments in a housing development has been increasing by a constant amount every year.

At the end of the first year the number of apartments was 150, and at the end of the sixth year the number of apartments was 600.

The number of apartments, y, can be determined by the equation $y = mt + n$, where t is the time, in years.

(a) Find the value of m. *[2]*

(b) State what m represents **in this context.** *[1]*

(c) Find the value of n. *[2]*

(d) State what n represents **in this context.** *[1]*

Working:

Answers:

(a) .....

(b) .....

.....

(c) .....

(d) .....

.....

15. Consider the functions $f(x)=x+1$ and $g(x)=3^x-2$.

(a) Write down

(i) the x-intercept of the graph of $y=f(x)$;

(ii) the y-intercept of the graph of $y=g(x)$. *[2]*

(b) Solve $f(x)=g(x)$. *[2]*

(c) Write down the interval for the values of x for which $f(x)>g(x)$. *[2]*

Working:

Answers:

(a) (i) .....

(ii) .....

(b) .....

(c) .....

14. [Maximum mark: 18]

A group of students at Dune Canyon High School were surveyed. They were asked which of the following products: books (B), music (M) or films (F), they downloaded from the internet.

The following results were obtained:

100 students downloaded music;
95 students downloaded films;
68 students downloaded films and music;
52 students downloaded books and music;
50 students downloaded films and books;
40 students downloaded all three products;
8 students downloaded books **only**;
25 students downloaded none of the three products.

(a) Use the above information to complete a Venn diagram. [5]

(b) Calculate the number of students who were surveyed. [2]

(c) (i) On your Venn diagram, shade the set $(F \cup M) \cap B'$. Do not shade any labels or values on the diagram.

(ii) Find $n\left((F \cup M) \cap B'\right)$. [3]

(d) A student who was surveyed is chosen at random.
Find the probability that

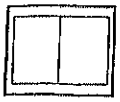
(i) the student downloaded music;

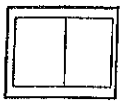
(ii) the student downloaded books, given that they had not downloaded films;

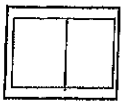
(iii) the student downloaded at least two of the products. [6]

Dune Canyon High School has 850 students.

(e) Find the expected number of students at Dune Canyon High School that downloaded music. [2]







17. [Maximum mark: 13]

The weights of sheep on a farm are normally distributed with a mean of 110 kg and a standard deviation of 8 kg.

(a) Sketch a diagram of the distribution of the weights of these sheep. On your diagram, label the mean and label one standard deviation above and below the mean. [2]

(b) (i) A sheep has a weight of 94 kg. Write down the number of standard deviations that this weight is below the mean.

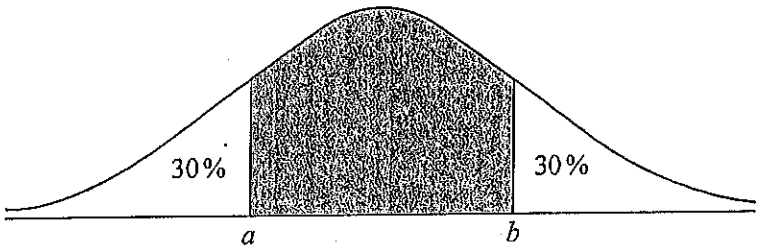
(ii) Find the probability that a sheep, chosen at random, weighs more than 94 kg. [3]

(c) (i) Find the probability that a sheep, chosen at random, weighs between 88 kg and 116 kg.

(ii) The farmer weighs 160 sheep. Find the number of sheep that he would expect to weigh between 88 kg and 116 kg. [4]

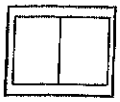
(d) Given that 75 % of the sheep weigh **less than** w kg, find the value of w. [2]

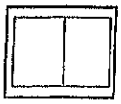
A sheep is chosen at random. Its weight is within the central shaded region of the following diagram.

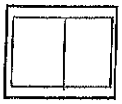


Weight of sheep

(e) Find the value of a and of b. [2]







18. [Maximum mark: 19]

Don took part in a project investigating wind speed, $x\,\text{km h}^{-1}$, and the time, y minutes, to fully charge a solar powered robot.

The investigation was carried out six times. The results are recorded in the table.

Wind Speed, x, (km h^{-1})	6	10	16	24	28	30
Time, y, (minutes)	28	26	30	33	38	37

(a) **On graph paper**, draw a scatter diagram to show the results of Don's investigation. Use a scale of 1 cm to represent 2 units on the x-axis, and 1 cm to represent 5 units on the y-axis. [4]

(b) Calculate

(i) $\overline{x}$, the mean wind speed;

(ii) $\overline{y}$, the mean time to fully charge the robot. [2]

M is the point with coordinates $(\overline{x}, \overline{y})$.

(c) Plot and label the point M on your scatter diagram. [2]

(d) (i) Calculate r, Pearson's product–moment correlation coefficient.

(ii) Describe the correlation between the wind speed and the time to fully charge the robot. [4]

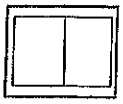
(e) (i) Write down the equation of the regression line y on x, in the form $y = mx + c$.

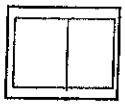
(ii) Draw this regression line on your scatter diagram.

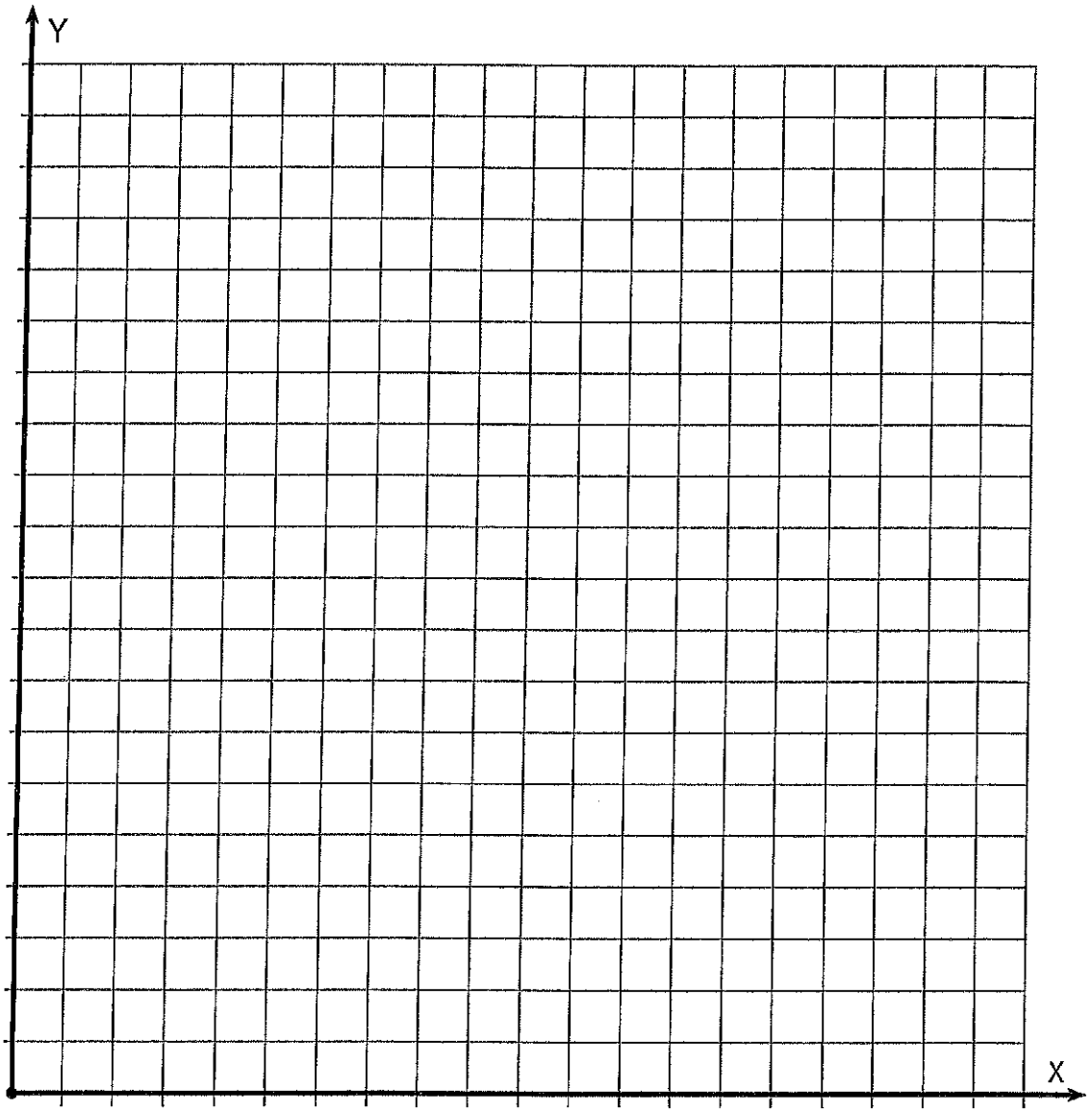
(iii) Hence or otherwise estimate the charging time when the wind speed is $27\,\text{km h}^{-1}$. [6]

Don concluded from his investigation: "There is no causation between wind speed and the time to fully charge the robot".

(f) In the context of the question, briefly explain the meaning of "no causation". [1]






Y
X

19. *[Maximum mark: 14]*

The Brahma chicken produces eggs with weights in grams that are normally distributed about a mean of 55 g with a standard deviation of 7 g. The eggs are classified as small, medium, large or extra large according to their weight, as shown in the table below.

Size	Weight (g)
Small	Weight < 53
Medium	$53 \leq$ Weight < 63
Large	$63 \leq$ Weight < 73
Extra Large	Weight ≥ 73

(a) Sketch a diagram of the distribution of the weight of Brahma chicken eggs. On your diagram, show clearly the boundaries for the classification of the eggs. *[3 marks]*

An egg is chosen at random.

(b) Find the probability that the egg is

(i) medium;

(ii) extra large. *[4 marks]*

There is a probability of 0.3 that a randomly chosen egg weighs more than w grams.

(c) Find w. *[2 marks]*

The probability that a Brahma chicken produces a large size egg is 0.121. Frank's Brahma chickens produce 2000 eggs each month.

(d) Calculate an estimate of the number of large size eggs produced by Frank's chickens each month. *[2 marks]*

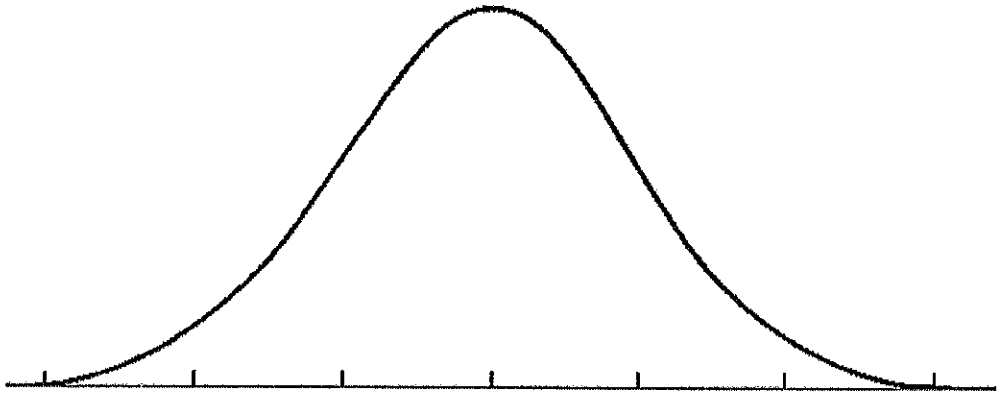
(Question ...ontinued)

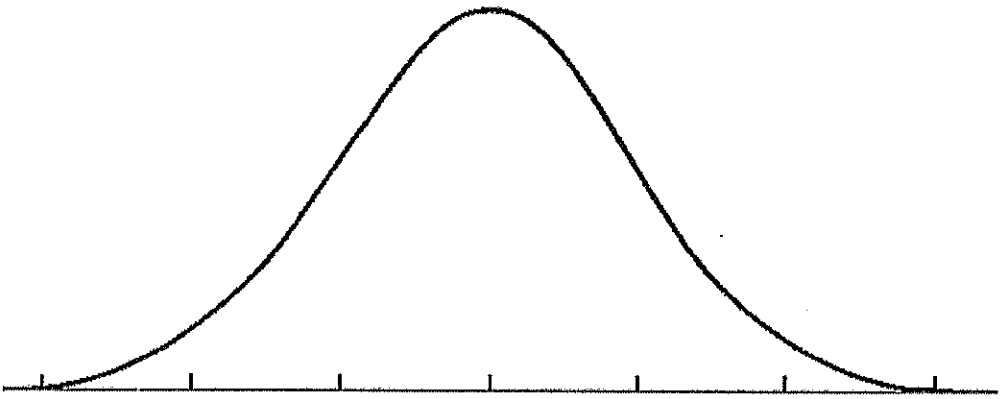
The selling price, in US dollars (USD), of each size is shown in the table below.

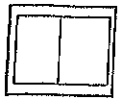
Size	Selling price (USD)
Small	0.30
Medium	0.50
Large	0.65
Extra Large	0.80

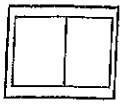
The probability that a Brahma chicken produces a small size egg is 0.388.

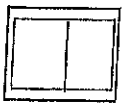
(e) Estimate the monthly income, in USD, earned by selling the 2000 eggs. Give your answer correct to two decimal places. *[3 marks]*











20. [Maximum mark: 14]

John purchases a new bicycle for 880 US dollars (USD) and pays for it with a Canadian credit card. There is a transaction fee of 4.2% charged to John by the credit card company to convert this purchase into Canadian dollars (CAD).

The exchange rate is 1 USD = 1.25 CAD.

(a) Calculate, in CAD, the total amount John pays for the bicycle. [3]

John insures his bicycle with a US company. The insurance company produces the following table for the bicycle's value during each year.

Year	Value of the bicycle (USD)
1st	880
2nd	704
3rd	563.20
...	...

The values of the bicycle form a geometric sequence.

(b) Find the value of the bicycle during the 5th year. **Give your answer to two decimal places.** [3]

(c) Calculate, in years, when the bicycle value will be less than 50 USD. [2]

During the 1st year John pays 120 USD to insure his bicycle. Each year the amount he pays to insure his bicycle is reduced by 3.50 USD.

(d) Find the total amount John has paid to insure his bicycle for the first 5 years. [3]

John purchased the bicycle in 2008.

(e) Justify why John should not insure his bicycle in 2019. [3]